LEVEL K

K 1-10 : Review of Linear Functions

Any linear function can be expressed in the form $y = mx + b$, where *m* is the gradient of the line, *b* is the *y*-intercept.

Given a line passes through (x_1, y_1) and (x_2, y_2), the gradient of the line can be found by:

$$gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

Line $x = k$ is a straight line **parallel to the *y*-axis**.

Line $y = k$ is a straight line **parallel to the *x*-axis**.

K 11-30 : Quadratic Functions & Graphs

Given a quadratic function of the form $y = a(x-p)^2 + q$, where $a \neq 0$

- The axis of symmetry is $x = p$
- The vertex is (p, q)

How to complete the square,

$$\begin{aligned} y &= 3x^2 + 7x + 1 \\ &= 3\left(x^2 + \frac{7}{3}x\right) + 1 \\ &= 3\left[\left(x + \frac{7}{6}\right)^2 - \frac{49}{36}\right] + 1 \\ &= 3\left(x + \frac{7}{6}\right)^2 - 3 \times \frac{49}{36} + 1 \\ &= 3\left(x + \frac{7}{6}\right)^2 - \frac{37}{12} \end{aligned}$$

The graph of $y = a(x-p)^2 + q$ is a translation of $y = ax^2$, p units along the *x*-axis and q units along the *y*-axis. The vertex is (p, q)

The graph of $y = 2(x+4)^2 - 5$ is a translation of $y = 2x^2$, -4 unit(s) along the x-axis, -5 unit(s) along the y-axis.

To draw the graph of a quadratic function, find the (1) vertex, (2) *y*-intercept and (3) *x*-intercept

$y = 2x^2 - 7x + 3$

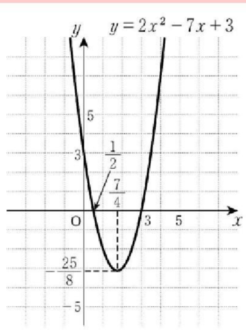
① The vertex is $\left(\frac{7}{4}, -\frac{25}{8}\right)$.

② If $x = 0$, then $y = 3$.

Therefore, the y-intercept is $(0, 3)$.

③ If $y = 0$, then $x = \frac{1}{2}, 3$.

Therefore, the x-intercepts are $\left(\frac{1}{2}, 0\right)$ and $(3, 0)$.



K 31-40 : Equations of Quadratic Functions

Form 1: Vertex form

$\boldsymbol{y = a(x-p)^2 + q}$, where (p, q) is the vertex of the parabola

If $a > 0$, the parabola opens upward.

If $a < 0$, the parabola opens downward.

Form 2: x-intercept form

$\boldsymbol{y = a(x-\alpha)(x-\beta)}$, where α and β are the zeros / *x*-intercepts of the quadratic function.

The parabola crosses the *x*-axis at $(\alpha, 0)$ and $(\beta, 0)$

Form 3: Standard form

$\boldsymbol{y = ax^2 + bx + c}$, where *c* is the *y*-intercept

To determine the equation of a quadratic function, we first identify the type of information given:

Type 1: Vertex & one point

The vertex is $(2, 1)$, and the parabola passes through point $(0, 5)$.

[Sol] Since the vertex is $(2, 1)$,

let $\boxed{y = a(x-2)^2 + 1}$

Because the parabola passes through $(0, 5)$,

$5 = 4a + 1$

Substituting $x = 0$ and $y = 5$ into the equation above.

Type 2: x-intercepts & one point

The parabola intersects the x-axis at points $(1, 0)$ and $(3, 0)$, and intersects the y-axis at point $(0, 6)$.

[Sol] Since the parabola intersects the x-axis at $(1, 0)$ and $(3, 0)$, let $\boxed{y = a(x-1)(x-3)}$

Since the parabola intersects the y-axis at $(0, 6)$,

$6 = 3a \qquad a = 2$

Type 3: Three points

The parabola passes through three points $(-1, 0), (2, 3)$ and $(3, -4)$.

[Sol] Let $\boxed{y = ax^2 + bx + c}$

Since the parabola passes through $(-1, 0), (2, 3)$ and $(3, -4)$,

$$\begin{cases} a - b + c = 0 & \cdots ① \\ 4a + 2b + c = 3 & \cdots ② \\ 9a + 3b + c = -4 & \cdots ③ \end{cases}$$

K 41-70 : Maxima & Minima of Quadratic Functions

Given a quadratic function $y = ax^2 + bx + c$

When $\boldsymbol{a > 0}$, the function has a **minimum** value at the vertex. When $\boldsymbol{a < 0}$, the function has a **maximum** value at the vertex.

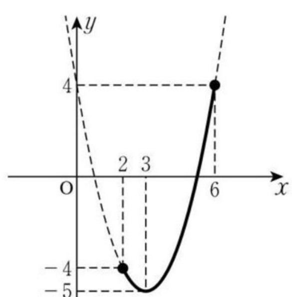
To find the minimum/maximum value:

1) Draw the graph of the function

2) Highlight only the part of the curve which lies within the domain

3) Range: Minimum value $\leq y \leq$ Maximum value

$y = x^2 - 6x + 4$ when $2 \leqslant x \leqslant 6$
$= (x-3)^2 - 5$



Maximum value: 4 , at $x = 6$

Minimum value: -5 , at $x = 3$

Range: $-5 \leqslant y \leqslant 4$

How to find the minimum/maximum value (when the domain is arbitrary, K51-60):

1) Prepare two rulers, pretend that they are the lower boundary and the upper boundary of the domain.

2) Place the rulers on the graph. Gradually move both rulers from left to right.

3) Observe how the maximum and minimum value changes as the ruler passes through certain "critical points". E.g. when ruler $x=a+4$ is to the left of $x=5$, the minimum value of the function is f(a+4), but when it moves to the right of x=5, the minimum value becomes f(5).

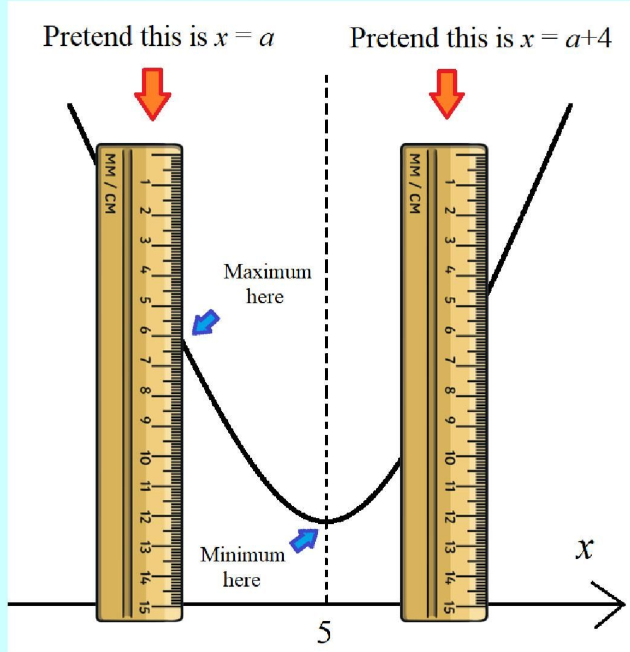
(when the function is arbitrary, K61-70):

1) As (1) above.

2) Place the rulers on the graph. Move both rulers from right to left. In that sense, the parabola is moving from left to right.

3) As (3) above.

Visualise the method:



K 71-80 : Quadratic Functions & Equations

The number of real roots of $ax^2 + bx + c = 0$ is the same as the number of common points of $\boldsymbol{y = ax^2 + bx + c}$ and the ***x*-axis**.

Given a quadratic function $y = ax^2 + bx + c$, and define $D = b^2 - 4ac$.

- 2 common points $\Leftrightarrow$ $D > 0$
- 1 common point $\Leftrightarrow$ $D = 0$
- 0 common points $\Leftrightarrow$ $D < 0$

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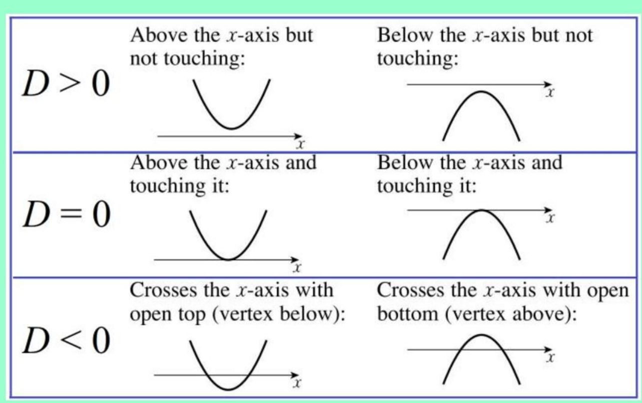
Similarly, the number of real solutions of the simultaneous equations $\begin{cases} y = ax^2 + bx + c \\ y = mx + n \end{cases}$ is the same as the number of common points of $y = ax^2 + bx + c$ and $y = mx + n$.

*** The quadratic equation would be $ax^2 + (b-m)x + c - n = 0$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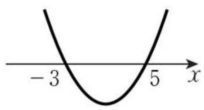
K 81-90 : Quadratic Functions & Inequalities

To solve a quadratic inequality, first find the x-intercept(s) of the quadratic function.

$x^2-2x-15>0$

[Sol] $(x+3)(x-5)>0$



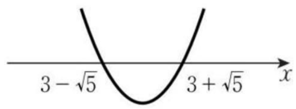
From the sketch,

$x<-3,\ x>5$

$x^2-6x+4<0$

[Sol] $x^2-6x+4=0$

$x=3\pm\sqrt{5}$



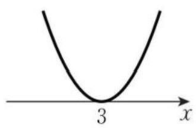
From the sketch,

$3-\sqrt{5}<x<3+\sqrt{5}$

$x^2-6x+9>0$

[Sol] $y=x^2-6x+9$

$=(x-3)^2$



From the sketch,

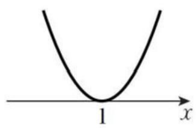
$y>0$ for all values of x, except when $x=3$.

$x<3,\ x>3\ (x\neq 3)$

$x^2-2x+1<0$

[Sol] $y=x^2-2x+1$

$=(x-1)^2$



From the sketch above, when $y<0$ there are no values of x.

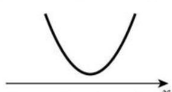
No solution

In some cases, the quadratic function does not touch /cross the x-axis ($D<0$).

$x^2-3x+4>0$

[Sol] Calculating the discriminant, D, of $x^2-3x+4=0$,

$D=9-16=-7<0$



From the sketch,

$y>0$ for all values of x.

All real numbers

$x^2-3x+4\geqslant 0$

[Sol] From the sketch,

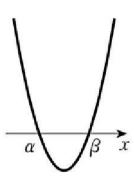
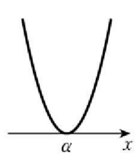
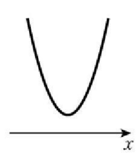
All real numbers

$x^2-3x+4<0$

[Sol] From the sketch, when $y<0$ there are no values of x.

No solution

<u>Summary:</u> (assume $a>0$)

The sign of D	$D>0$	$D=0$	$D<0$
The graph of $y=ax^2+bx+c$			
Solution of:			
$ax^2+bx+c>0$	$x<\alpha,\ x>\beta$	$x<\alpha,\ x>\alpha$	All real numbers
$ax^2+bx+c\geqslant 0$	$x\leqslant\alpha,\ x\geqslant\beta$	All real numbers	All real numbers
$ax^2+bx+c<0$	$\alpha<x<\beta$	No solution	No solution
$ax^2+bx+c\leqslant 0$	$\alpha\leqslant x\leqslant\beta$	$x=\alpha$	No solution

K 91-100 : Solutions of Quadratic Equations

Given a quadratic equation $ax^2+bx+c=0$ ($a>0$) with two solutions α and β ($\alpha<\beta$), we can determine the signs of α and β by determining the signs of D, $-\frac{b}{2a}$ and y-intercept, $f(0)$.

(i) When $D>0$, $-\frac{b}{2a}>0$, $f(0)>0$, then $\alpha>0,\ \beta>0$.

(ii) When $D>0$, $-\frac{b}{2a}<0$, $f(0)>0$, then $\alpha<0,\ \beta<0$.

(iii) When $f(0)<0$, then $\alpha<0,\ \beta>0$.

Given quadratic function $f(x)=ax^2+bx+c$ in the domain $h<x<k$,

$f(h)\cdot f(k)<0 \Rightarrow$ there is exactly one solution between h and k

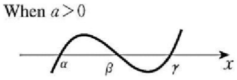
A general theorem (Intermediate Value Theorem) will be revisited in level N.

K 101-110 : Higher Degree Functions

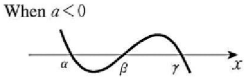
A function that can be written as $y=ax^3+bx^2+cx+d$ is called a cubic function.

- The possible sketches of the cubic function $y=a(x-\alpha)(x-\beta)(x-\gamma)$ ($\alpha<\beta<\gamma$) are:

When $a>0$

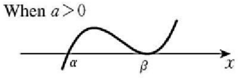


When $a<0$

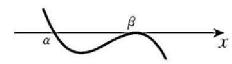


- The possible sketches of the cubic function $y=a(x-\alpha)(x-\beta)^2$ ($\alpha<\beta$) are:

When $a>0$

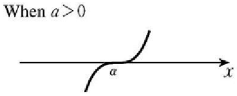


When $a<0$

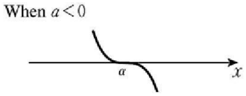


- The possible sketches of the cubic function $y=a(x-\alpha)^3$ are:

When $a>0$



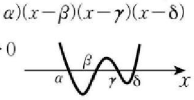
When $a<0$



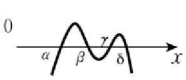
A function that can be written as $y=ax^4+bx^3+cx^2+dx+e$ is called a quartic function.

- The possible sketches of the quartic function $y=a(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$ ($\alpha<\beta<\gamma<\delta$) are:

When $a>0$

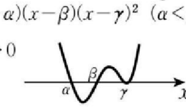


When $a<0$

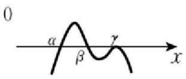


- The possible sketches of the quartic function $y=a(x-\alpha)(x-\beta)(x-\gamma)^2$ ($\alpha<\beta<\gamma$) are:

When $a>0$

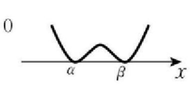


When $a<0$

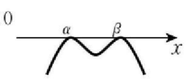


- The possible sketches of the quartic function $y=a(x-\alpha)^2(x-\beta)^2$ ($\alpha<\beta$) are:

When $a>0$

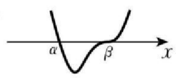


When $a<0$

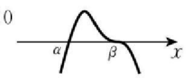


- The possible sketches of the quartic function $y=a(x-\alpha)(x-\beta)^3$ $(\alpha<\beta)$ are:

When $a>0$

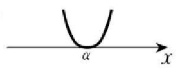


When $a<0$

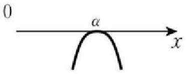


- The possible sketches of the quartic function $y=a(x-\alpha)^4$ are:

When $a>0$



When $a<0$



<u>Extra Notes:</u>

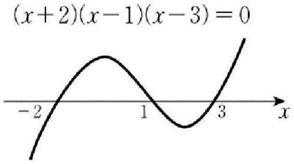
The graph of $y=a(x-p)^3+q$ is a translation of $y=ax^3$, p units along the x-axis and q units along the y-axis.

Similarly, the graph of $y=b(x-p)^4+q$ is a translation of $y=bx^4$, p units along the x-axis and q units along the y-axis.

K 111-120 : Higher Degree Equations & Inequalities

To solve higher degree equations, we factorise the polynomial expression and equate to zero.

$(x+2)(x-1)(x-3)=0$



$y=(x+2)(x-1)(x-3)$

$x=-2,\ 1,\ 3$

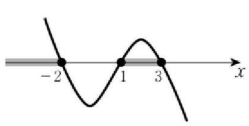
To solve higher degree inequalities, sketch the graph of the function and determine the range of values of x which satisfies the inequality.

$(x+2)(x-1)(3-x)\geqslant 0$

[Sol] $-(x+2)(x-1)(x-3)\geqslant 0$

Sketching

$y=-(x+2)(x-1)(x-3)$,



Therefore, $x\leqslant -2,\ 1\leqslant x\leqslant 3$

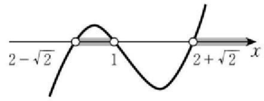
$(x-1)(x^2-4x+2)>0$

[Sol] Solving $x^2-4x+2=0$,

$x=2\pm\sqrt{2}$

Sketching

$y=(x-1)(x^2-4x+2)$,



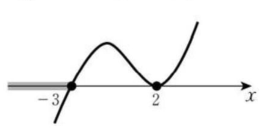
Therefore,

$2-\sqrt{2}<x<1,\ x>2+\sqrt{2}$

$(x+3)(x-2)^2\leqslant 0$

[Sol] Sketching

$y=(x+3)(x-2)^2$,



Therefore,

$x\leqslant -3,\ x=2$

$(x+2)(x^2+3x+5)>0$

[Sol] Let D be the discriminant of the equation $x^2+3x+5=0$.

As $D=9-20=-11<0$,

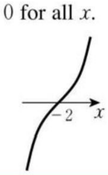
so $x^2+3x+5>0$ for all x.

Therefore,

$x+2>0$

Thus,

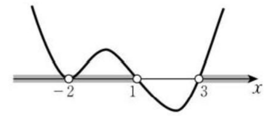
$x>-2$



$(x+2)^2(x-1)(x-3)>0$

[Sol] Sketching

$y=(x+2)^2(x-1)(x-3)$,



Therefore,

$x<-2,\ -2<x<1,\ x>3$

$(x+2)(x-1)(x^2+3x+3)<0$

[Sol]

Calculating the discriminant, D, of $x^2+3x+3=0$,

$D=9-12=-3<0$.

So $x^2+3x+3>0$ for all x.

Therefore,

$(x+2)(x-1)<0$

Thus, $-2<x<1$

<u>Note:</u> One may need to apply the factor theorem.

K 121-140 : Graphs of Fractional Functions

The asymptotes of the graph of $\boldsymbol{y=\frac{k}{x-p}+q}$ $(k\neq 0)$ are $\boldsymbol{x=p}$ and $\boldsymbol{y=q}$.

$y=\dfrac{x+1}{x-2}$

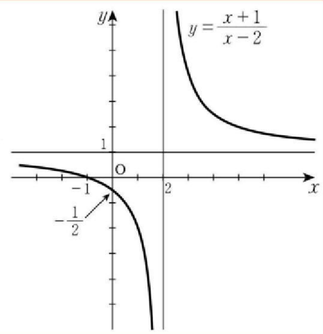
$y=1+\dfrac{3}{x-2}$

$$\begin{array}{r} 1 \\ x-2\,\overline{)\,x+1} \\ x-2 \\ \hline 3 \end{array}$$

- Asymptotes: $x=2,\ y=1$
- Intercepts:

x-axis: $(-1,\ 0)$

y-axis: $\left(0,\ -\dfrac{1}{2}\right)$



<u>Extra Notes:</u>

Given a fractional function $y=\dfrac{k}{x-p}+q$ $(k\neq 0)$, the graph's asymptotes are $x=p$ and $y=q$. This graph has been translated from $y=\dfrac{k}{x}$, p units along the x-axis and q units along the y-axis.

If a domain is specified, highlight only the part of the graph which lies within the domain.

$f(x)=\dfrac{2x-5}{x-3}$ $\boxed{(2\leqslant x\leqslant 6,\text{ but with } x\neq 3)}$

[Sol] $f(x)=2+\dfrac{1}{x-3}$

- Asymptotes:

$x=3,\ y=2$

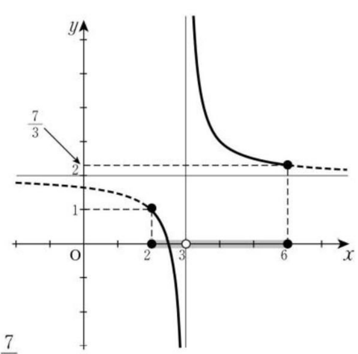
- Intercepts:

x-axis: $\left(\dfrac{5}{2},\ 0\right)$

y-axis: $\left(0,\ \dfrac{5}{3}\right)$

- Range:

$f(2)=1$, $f(6)=\dfrac{7}{3}$



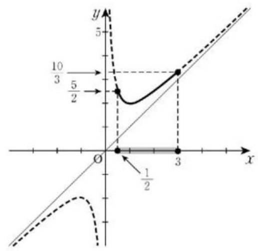
From the graph, $f(x)\leqslant 1$, $f(x)\geqslant \dfrac{7}{3}$

The asymptotes of the graph of $\boldsymbol{y=ax+\frac{b}{x}}$ are $\boldsymbol{x=0}$ and $\boldsymbol{y=ax}$ (oblique asymptote).

Given a fractional function $\boldsymbol{y = x - p + \frac{k}{x-p} + q}$.

The graph has been translated from $\boldsymbol{y = x + \frac{k}{x}}$, p units along the x-axis and q units along the y-axis.

Given the fractional function $f(x) = x + \frac{1}{x}$ $\left(\frac{1}{2} \leqslant x \leqslant 3\right)$, draw the graph and find the maximum value.



[Sol] $f\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{5}{2}$

$f(3) = 3 + \frac{1}{3} = \frac{10}{3}$

From the graph:

Maximum value: $f(3) = \frac{10}{3}$

K 141-150 : Fractional Equations & Inequalities

To solve fractional equations,

$$\frac{x+4}{x+2} = \frac{x}{3}$$

[Sol] Multiplying both sides by $3(x+2)$,

$3(x+4) = x(x+2)$ (where $x \neq -2$)

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4, -3$

We must always check each solution and reject any that make a denominator in the equation zero.

$$\frac{1}{x-1} - \frac{2}{x+1} = \frac{x}{x-1}$$

[Sol] Multiplying both sides of the equation by $(x-1)(x+1)$,

$x + 1 - 2(x-1) = x(x+1)$ (where $x \neq 1$, $x \neq -1$)

$x^2 + 2x - 3 = 0$

$x = -3, 1$

$x = 1$ is invalid. ☜ Since the denominator is 0 when $x = 1$.

Therefore, $x = -3$

To solve a fractional inequality,

<u>Method 1:</u> Graphical method

$$\frac{(x+1)(x-2)}{x} \leqslant 0$$

[Sol] Rewriting, $x - 1 - \frac{2}{x} \leqslant 0$,

$x - 1 \leqslant \frac{2}{x}$

Solving $x - 1 = \frac{2}{x}$,

$x(x-1) = 2$ (where $x \neq 0$)

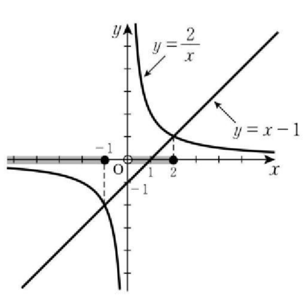
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

From the graph,

$x \leqslant -1$, $0 < x \leqslant 2$



<u>Method 2:</u> Multiply the square of the denominator

$$\frac{5}{x+2} \geqslant \frac{2}{x-1}$$

[Sol] $\frac{5}{x+2} - \frac{2}{x-1} \geqslant 0$ ⇨ $\frac{5(x-1) - 2(x+2)}{(x+2)(x-1)} \geqslant 0$

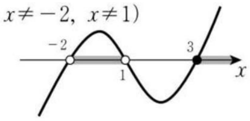
$\frac{3(x-3)}{(x+2)(x-1)} \geqslant 0$

Multiplying both sides by $(x+2)^2(x-1)^2$,

$3(x-3)(x+2)(x-1) \geqslant 0$ (where $x \neq -2$, $x \neq 1$)

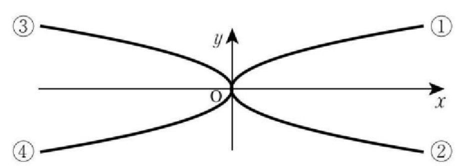
From the graph,

$-2 < x < 1$, $x \geqslant 3$



K 151-160 : Graphs of Irrational Functions

- Given an irrational function of the form $y = \sqrt{k(x-p)} + q$, the domain and range of the function are as follows:
  When $k > 0$, Domain: $x \geqslant p$, Range: $y \geqslant q$
  When $k < 0$, Domain: $x \leqslant p$, Range: $y \geqslant q$
- An irrational function of the form $y = \sqrt{k(x-p)} + q$ is a translation of $y = \sqrt{kx}$, p units along the x-axis, and q units along the y-axis.
- The graphs of some common irrational functions are shown below.
  ① $y = \sqrt{x}$, ② $y = -\sqrt{x}$, ③ $y = \sqrt{-x}$, ④ $y = -\sqrt{-x}$



K 161-170 : Irrational Equations & Inequalities

When solving irrational equation, check if solutions correspond to common points on the graph.

$$\sqrt{4-x} = x - 2$$

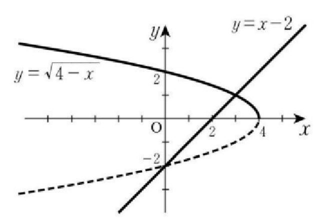
[Sol] Squaring both sides,

$4 - x = (x-2)^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3$



From the graph, $x = 0$ is an extraneous solution.

Therefore, $x = 3$

Alternatively, substitute the solution into the original equation and check if LHS = RHS.

Solve the irrational equation $\sqrt{5-2x} = -x + 1$.

[Sol] Squaring both sides,

$5 - 2x = (-x+1)^2$ ⇨ $x^2 = 4$ ⇨ $x = \pm 2$

Substituting into the original equation,

(i) When $x = 2$, LHS $= 1$, RHS $= -1$

Therefore, $x = 2$ is an extraneous solution.

(ii) When $x = -2$, LHS $= 3$, RHS $= 3$

From (i) and (ii), $x = -2$

To solve irrational inequalities, first solve the equation, then look for range of values of x that satisfy the inequality from the **graph**.

$\sqrt{2x+1} > x-1$

[Sol] Let $\sqrt{2x+1} = x-1$ — First, solve the equation.

Squaring both sides,

$2x+1 = (x-1)^2$

$x^2 - 4x = 0$

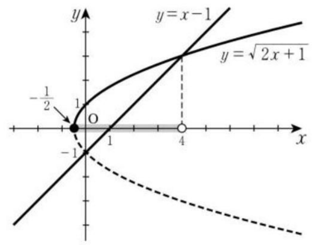
$x(x-4) = 0$

$x = 0,\ 4$

From the graph, $x = 0$ is an extraneous solution.

Thus, $x = 4$

Therefore, $-\frac{1}{2} \leqslant x < 4$ — For these values of x, the graph of $y = \sqrt{2x+1}$ lies above the graph of $y = x-1$.



K 171-190 : Exponential Functions and Graphs

Properties of exponents:

Laws of Exponents I

When $a \neq 0,\ b \neq 0$,

$a^0 = 1$ $\quad a^m \times a^n = a^{m+n}$ $\quad (a^m)^n = a^{mn}$

$a^{-n} = \frac{1}{a^n}$ $\quad a^m \div a^n = a^{m-n}$ $\quad (ab)^m = a^m b^m$

Laws of Exponents II

When $a > 0$, $b > 0$ and m, n and p are positive integers:

$\sqrt[n]{a^m} = (\sqrt[n]{a}\,)^m$ $\quad \sqrt[mp]{a^{np}} = \sqrt[m]{a^n}$ $\quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$ $\quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Laws of Exponents III

When $a > 0$, m is an integer, and n is a positive integer,

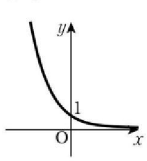
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $\quad (a^{\frac{1}{n}} = \sqrt[n]{a}\ )$

The graph of an exponential function:

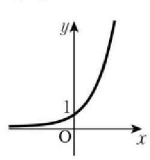
When $a > 0$ and $a \neq 1$, a function of the form $y = a^x$ is called an exponential function of x, where a is the base.

The graph of $y = a^x$ passes through point $(0,\ 1)$, and the x-axis is the asymptote.

When $0 < a < 1$



When $a > 1$



Transformation effects:

$\boldsymbol{y = \left(\frac{1}{a}\right)^x}$ **or** $\boldsymbol{y = a^{-x}}$ (where $a > 0$ and $a \neq 1$) is a reflection of $\boldsymbol{y = a^x}$ with respect to the $\boldsymbol{y}$**-axis**.

$\boldsymbol{y = -a^x}$ is a reflection of $\boldsymbol{y = a^x}$ with respect to the $\boldsymbol{x}$**-axis**.

The graph of $\boldsymbol{y = a^{x-p} + q}$ (where $a > 0$ and $a \neq 1$) is a translation of $\boldsymbol{y = a^x}$, p units along the x-axis and q units along the y-axis.

Since an exponential function is monotonic,

Given the exponential function $y = a^x$

★ $a > 1$, then as x increases, y increases.
Therefore, $p < q$ if and only if $a^p < a^q$.

★ $0 < a < 1$, then as x increases, y decreases.
Therefore, $p < q$ if and only if $a^p > a^q$.

Given the exponential functions $y = a^x$ and $y = b^x$, when $a > 0$, $b > 0$ and $p > 0$, $a < b$ if and only if $a^p < b^p$.

K 191-200 : Exponential Equations & Inequalities

When solving equations, use a common base.

$8^x = 2^{x+1}$

[Sol] $2^{3x} = 2^{x+1}$

$3x = x+1$

$2x = 1$

$x = \frac{1}{2}$

$9^x = 3\sqrt{3}$

[Sol] $3^{2x} = 3^{\frac{3}{2}}$

$2x = \frac{3}{2}$

$x = \frac{3}{4}$

$2^{2x+1} + 3\cdot 2^x - 2 = 0$

[Sol]

$2(2^x)^2 + 3\cdot 2^x - 2 = 0$

Let $2^x = X$ (where $X > 0$)

$2X^2 + 3X - 2 = 0$

$X = \frac{1}{2},\ -2$

Since $X > 0$, $X = -2$ is an extraneous solution.

Therefore, $X = \frac{1}{2}$

$x = -1$

When solving inequalities, we switch the inequality sign if the base is < 1, leave the inequality sign unchanged if the base is > 1.

$9^x < 27$

[Sol] $3^{2x} < 3^3$

$2x < 3$

$x < \frac{3}{2}$

$\left(\frac{1}{4}\right)^x < \frac{1}{8}$

[Sol] $\left(\frac{1}{2}\right)^{2x} < \left(\frac{1}{2}\right)^3$

$2x > 3$

$x > \frac{3}{2}$

$2^{2x+1} + 3\cdot 2^x - 2 < 0$

[Sol]

$2(2^x)^2 + 3\cdot 2^x - 2 < 0$

Let $2^x = X$ (where $X > 0$)

$2X^2 + 3X - 2 < 0$

$(X+2)(2X-1) < 0$

$-2 < X < \frac{1}{2}$

Since $X > 0$,

$0 < X < \frac{1}{2}$

$0 < 2^x < \frac{1}{2}$

$x < -1$